

Performance Analysis of Matrix Inversion Algorithms for Massive MIMO Precoding under Rural and Urban Scenarios

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ABSTRACT

Massive MIMO or Very Large Scale MIMO is a key technology in recent 5G and beyond 5G wireless standards to attain high speed, secure, error free communication and accessibility among trillions of users. All the above is possible with increase in complexity of the massive MIMO system. To improve massive MIMO system performance, efficient linear precoding algorithms are used at the downlink. The complexity of linear precoding algorithms lies on the hardware demanding large size matrix inversion module. In this paper, the performance of various approximations such as Neumann series, Conjugate Gradient (CG) and Preconditioned Conjugate Gradient (PCG) algorithms that simplify the matrix inversions of linear precoding algorithms are discussed. The simulations are carried out under various propagation scenarios like rural macro cell and urban micro cell.

The future wireless systems need to provide many developments over fixed networks in terms of security, speed, robustness and variety of services being offered. It is possible to provide the above services to the everyday increasing number of customers only by means of technology that supports a very high capacity, data rate, low power consumption and reduced latency. Though, these targets cannot be satisfied simultaneously, they form the basis for the higher capacity 5G networks. One of many techniques used to meet the above desired benefits is Multiple Input Multiple Output (MIMO). In Point to Point MIMO, both Base Station (BS) and Mobile Terminal (MT) are equipped with multiple antennas that exploit the spatial diversity property of the channel. In Multi-User MIMO (MU-MIMO) systems, a central BS is connected to more than one mobile terminals, thereby reducing the complexity of MT s. An emerging area in MUMIMO communication is Massive MIMO systems. In Massive MIMO, antennas of around hundred are installed in BS whereas MU-MIMO employs antennas in the order of 10 to 20 inside BS.

Keywords—Massive MIMO, Zero-Forcing precoding, Krylov subspace iterative algorithms, Conjugate gradient precoding, Neumann series

1. INTRODUCTION

The future wireless systems need to provide many developments over fixed networks in terms of security, speed, robustness and variety of services being offered. It is possible to provide the above services to the everyday increasing number of customers only by means of technology that supports a very high capacity, data rate, low power consumption and reduced latency. Though, these targets cannot be satisfied simultaneously, they form the basis for the higher capacity 5G networks [1-4]. One of many techniques used to meet the above desired benefits is Multiple Input Multiple Output (MIMO). In Point to Point MIMO, both Base Station (BS) and Mobile Terminal (MT) are equipped with multiple antennas that exploit the spatial diversity property of the channel. In Multi-User MIMO (MU-MIMO) systems, a central BS is connected to more than one mobile terminals, thereby reducing the complexity of MT s. An emerging area in MUMIMO communication is Massive MIMO systems. In Massive MIMO, antennas of around hundred are installed in BS whereas MU-MIMO employs antennas in the order of 10 to 20 inside BS. The channel noise effects and channel fast fading effects diminishes. But, it causes interference between the adjacent users which is mitigated by means

of efficient precoding techniques. This precoding is an efficient signal processing technique done on modulated symbols at the transmitter side. By the application of suitable precoding techniques, full potential massive MIMO system can be obtained. Precoding is a signal processing technique that transforms the input multiuser data into the array of data vectors of size equal to the antenna array dimension of the massive MIMO BS transmitter. For regular MIMO systems any type of precoding algorithm such as linear and nonlinear techniques is used as the size of matrix is small. Dirty Paper Coding (DPC) and THP precoding are the non linear precoding techniques discussed in [5] and [6] that achieve optimum capacity. But, such complex non-linear algorithms fail for massive MIMO system as their dimension is very huge in the order of Hundreds. Hence, linear precoding techniques such as Matched Filtering (MF), Regularized Zero Forcing (RZF), Zero Forcing (ZF) are the linear precoding techniques that achieve satisfactory performance with lesser complexity compared with non-linear precoding techniques. But the implementation of linear precoding techniques depends on the ability to perform matrix inverse operations faster though the dimension of the antenna array increases with the matrix size. Moreover, hardware implementations find it difficult to implement matrix inversions as they deal with division operations and implementing division in hardware is very tedious. Since, finding matrix inversions directly is challenging in hardware suitable approximations can be used to replace direct inversion operation with tolerable error performance.

2. LITERATURE SURVREY

More precisely, on a quasistatic channel where a codeword spans across only one time and frequency coherence interval, the reliability of a point-to-point MIMO link scales according to Prob (link outage) $\sim \text{SNR}^{-n_t n_r}$ - where n_t and n_r are the numbers of transmit and receive antennas, respectively, and signal-to-noise ratio is denoted by SNR. On a channel that varies rapidly as a function of time and frequency, and where circumstances permit coding across many channel coherence intervals, the achievable rate scales as $\log(1 + \text{SNR})$. The gains in multiuser systems are even more impressive, because such systems offer the possibility to transmit simultaneously to several users and the flexibility to select what users to schedule for reception at any given point in time [2]. The price to pay for MIMO is increased complexity of the hardware [number of radio frequency (RF) chains] and the complexity and energy consumption of the signal processing at both ends. For point-to-point links, complexity at the receiver is usually a greater concern than complexity at the transmitter. For example, the complexity of optimal signal detection alone grows exponentially with n_t [3], [4]. In multiuser systems, complexity at the transmitter is also a concern since advanced coding schemes must often be used to transmit information simultaneously to more than one user while maintaining a controlled level of interuser interference. Of course, another cost of MIMO is that of the physical space needed to accommodate the antennas, including rents of real estate. With very large MIMO, we think of systems that use antenna arrays with an order of magnitude more elements than in systems being built today, say 100 antennas or more. Very large MIMO entails an unprecedented number of antennas simultaneously serving a much smaller number of terminals. The disparity in number emerges as a desirable operating condition and a practical one as well. The number of terminals that can be simultaneously served is limited, not by the

number of antennas, but rather by our inability to acquire channel-state information for an unlimited number of terminals. Larger numbers of terminals can always be accommodated by combining very large MIMO technology with conventional time- and frequency-division multiplexing via orthogonal frequency-division multiplexing (OFDM). Very large MIMO arrays is a new research field both in communication theory, propagation, and electronics and represents a paradigm shift in the way of thinking both with regards to theory, systems, and implementation. The ultimate vision of very large MIMO systems is that the antenna array would consist of small active antenna units, plugged into an (optical) fieldbus. We foresee that in very large MIMO systems, each antenna unit uses extremely low power, in the order of milliwatts. At the very minimum, of course, we want to keep total transmitted power constant as we increase n_t , i.e., the power per antenna should be $\propto 1/n_t$. But in addition we should also be able to back off on the total transmitted power. For example, if our antenna array were serving a single terminal, then it can be shown that the total power can be made inversely proportional to n_t , in which case the power required per antenna would be $\propto 1/n_t^2$. Of course, several complications will undoubtedly prevent us from fully realizing such optimistic power savings in practice: the need for multiuser multiplexing gains, errors in channel state information (CSI), and interference. Even so, the prospect of saving an order of magnitude in transmit power is important because one can achieve better system performance under the same regulatory power constraints. Also, it is important because the energy consumption of cellular base stations is a growing concern. As a bonus, several expensive and bulky items, such as large coaxial cables, can be eliminated altogether. (The coaxial cables used for tower-mounted base stations today are up to 4 cm in diameter!) Moreover, very-large MIMO designs can be made extremely robust in that the failure of one or a few of the antenna units would not appreciably affect the system. Malfunctioning individual antennas may be hotswapped. The contrast to classical array designs, which use few antennas fed from a highpower amplifier, is significant. So far, the large-number-of-antennas regime, when n_t and n_r grow without bound, has mostly been of pure academic interest, in that some asymptotic capacity scaling laws are known for ideal situations. More recently, however, this view is changing, and a number of practically important system aspects in the large- (n_t, n_r) regime have been discovered. For example, [5] showed that asymptotically as $n_t \rightarrow \infty$ and under realistic assumptions on the propagation channel with a bandwidth of 20 MHz, a time-division multiplexing cellular system may accommodate more than 40 singleantenna users that are offered a net average throughput of 17 Mb/s both in the reverse (uplink) and the forward (downlink) links, and a throughput of 3.6 Mb/s with 95% probability! These rates are achievable without cooperation among the base stations and by relatively rudimentary techniques for CSI acquisition based on uplink pilot measurements. Several things happen when MIMO arrays are made large. First, the asymptotics of random matrix theory kick in. This has several consequences. Things that were random before, now start to look deterministic. For example, the distribution of the singular values of the channel matrix approaches a deterministic function [6]. Another fact is that very tall or very wide matrices tend to be very well conditioned. Also, when dimensions are large, some matrix operations such as inversions can be done fast, by using series expansion techniques (see the sidebar). In the limit of an infinite number of antennas at the base station, but with a single antenna per user, then linear processing in the form of maximumratio combining for the uplink (i.e., matched filtering with the channel vector, say \mathbf{h}) and maximum-

ratio transmission (beamforming with $\mathbf{h} \mathbf{h}^H$) \mathbf{H} on the downlink is optimal. This resulting processing is reminiscent of time reversal (TR), a technique used for focusing electromagnetic or acoustic waves [7], [8]. The second effect of scaling up the dimensions is that thermal noise can be averaged out so that the system is predominantly limited by interference from other transmitters. This is intuitively clear for the uplink, since coherent averaging offered by a receive antenna array eliminates quantities that are uncorrelated between the antenna elements, that is, thermal noise in particular. This effect is less obvious on the downlink, however. Under certain circumstances, the performance of a very large array becomes limited by interference arising from reuse of pilots in neighboring cells. In addition, choosing pilots in a smart way does not substantially help as long as the coherence time of the channel is finite. In a time-division duplex (TDD) setting, this effect was quantified in [5], under the assumption that the channel is reciprocal and that the base stations estimate the downlink channels by using uplink received pilots. Finally, when the aperture of the array grows, the resolution of the array increases. This means that one can resolve individual scattering centers with unprecedented precision. Interestingly, as we will see later on, the communication performance of the array in the large-number-of-antennas regime depends less on the actual statistics of the propagation channel but only on the aggregated properties of the propagation such as asymptotic orthogonality between channel vectors associated with distinct terminals. Of course, the number of antennas in a practical system cannot be arbitrarily large owing to physical constraints. Eventually, when letting n_r or n_t tend to infinity, our mathematical models for the physical reality will break down. For example, the aggregated received power would at some point exceed the transmitted power, which makes no physical sense. But long before the mathematical models for the physics break down, there will be substantial engineering difficulties. So, how large is “infinity” in this article? The answer depends on the precise circumstances of course, but in general, the asymptotic results of random matrix theory are accurate even for relatively small dimensions (even ten or so). In general, we think of systems with at least 100 antennas at the base station, but probably fewer than 1,000. Taken together, the arguments presented motivate entirely new theoretical research on signal processing and coding and network design for very large MIMO systems.

This article will survey some of these challenges. In particular, we will discuss ultimate information-theoretic performance limits, some practical algorithms, influence of channel properties on the system, and practical constraints on the antenna arrangements.

3. PROPOSED METHOD

We studied the performance of several main Linear Precoding algorithms in large-scale MIMO systems. For the convenience of expression, in a single cell system, we use \mathbf{H} to indicate the downlink channel matrix from base station to user. Based on the theoretical analysis, we studied the performance of the main Linear Precoding algorithms, and made performance simulation under the actual scene conditions, and compared with the theoretical results. In the single cell large-scale MIMO transmitter block diagram shown in Fig. 1, the base station precodes the signal and sends the signal vector to the user. \mathbf{s} represents the original signal, and \mathbf{x} represents the information vector sent by the sender to the user after precoding.

$$\mathbf{x} = \sqrt{\rho} \mathbf{W} \mathbf{s}$$

P is the average transmission power of the base station. Therefore, the signal received by the kth user in the cell can be expressed as:

$$y_k = Hx + n = \sqrt{\rho}H_k W_k S_k + n_k$$

Here, is the superposition of user interference signal and channel noise of the same pilot in other cells.

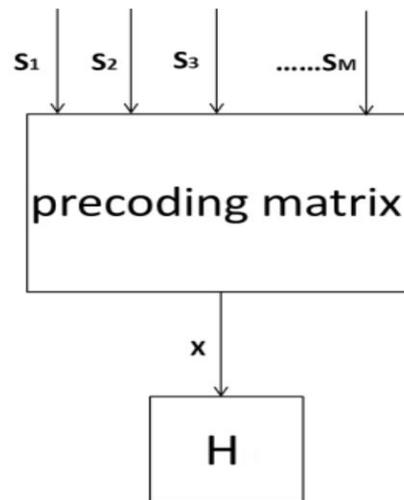


Fig 4: Single cell large-scale MIMO transmitter block diagram

Zero Forcing Precoding Algorithm

Zero forcing linear precoding scheme was originally proposed by Freescale Semiconductor Company. Different from the MRT precoding technology, ZERO FORCING precoding can completely remove the interference among users. It requires that all the signals received by users in the system do not contain the interference generated by other users, that is, make the precoding vector w_k of user K in the channel matrix of other users in the zero space of, that is, the interference items of other users in the signals received by user K:

$$\sum_{i=1, i \neq k}^k h_k w_i s = 0$$

The specific implementation process of Zero forcing linear precoding is as follows:

- 1)the channel is estimated in the client. That is to say, the pilot signal is used to estimate the channel among users and the estimated value of channel matrix is obtained (k=1,2,.....K)
- 2)feedback channel estimation of the client. The channel matrix estimated above (k=1,2,.....) is used to calculate the precoding matrix. In the TDD system, the base station directly estimates the channel information state of the downlink channel transmitter at the uplink pilot, and improves the accuracy of the channel information state; in the frequency division duplex system, the base station needs to obtain the channel information state of the transmitter through

the uplink feedback channel.

3)the precoding matrix is calculated at the transmitter. Zero forcing precoding can be expressed as pseudo inverse matrix of user channel matrix:

$$W_k = H_k^H (H_k H_k^H)^{-1}$$

It can be seen from the above formula that precoding in multiuser MIMO can be regarded as a process of maximizing the ratio of target user gain to inter user interference plus noise to some extent. MRT maximizes the target user's signal. When the interference between users is negligible compared with the noise, MRT is a near optimal algorithm in the signal limited system. Zero forcing precoding is intended to cancel the interference between users and lose some signal gain at the same time. When the number of users is large or the noise is relative to the interference, it can get the performance close to the system capacity limit. The main problem of Zero forcing precoding is that according to its scheme, antenna data must be processed together at the same time, and each antenna cannot be processed separately.

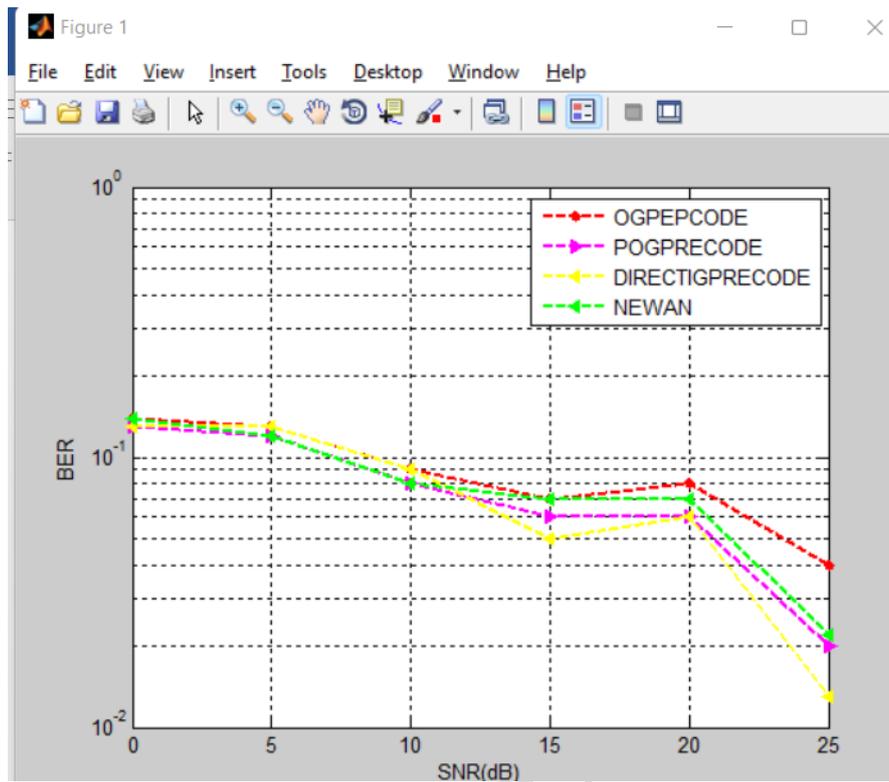
Conjugate Gradient Approximation

Conjugate Gradient (CG) is Krylov subspace based iterative solver for linear equations is described in [11]-[13]. Specifically, CG is a solver of problems is

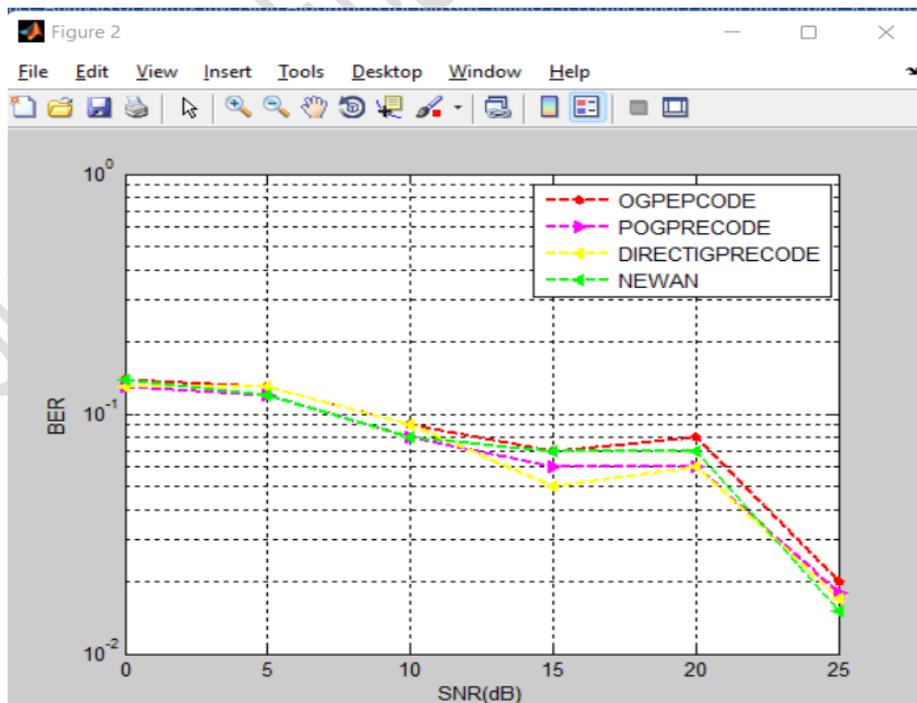
$$\hat{d} = \arg \min \|b - Gd\|$$

where G is a positive definite matrix and b is a column vector containing the transmitted symbol vector. This reduces the computational complexity to a considerable level instead of directly inverting the matrix . The result obtained at the end of final iteration is the product of inverted matrix and the data vector, is then multiplied with the channel matrix that yields the received signal vector.

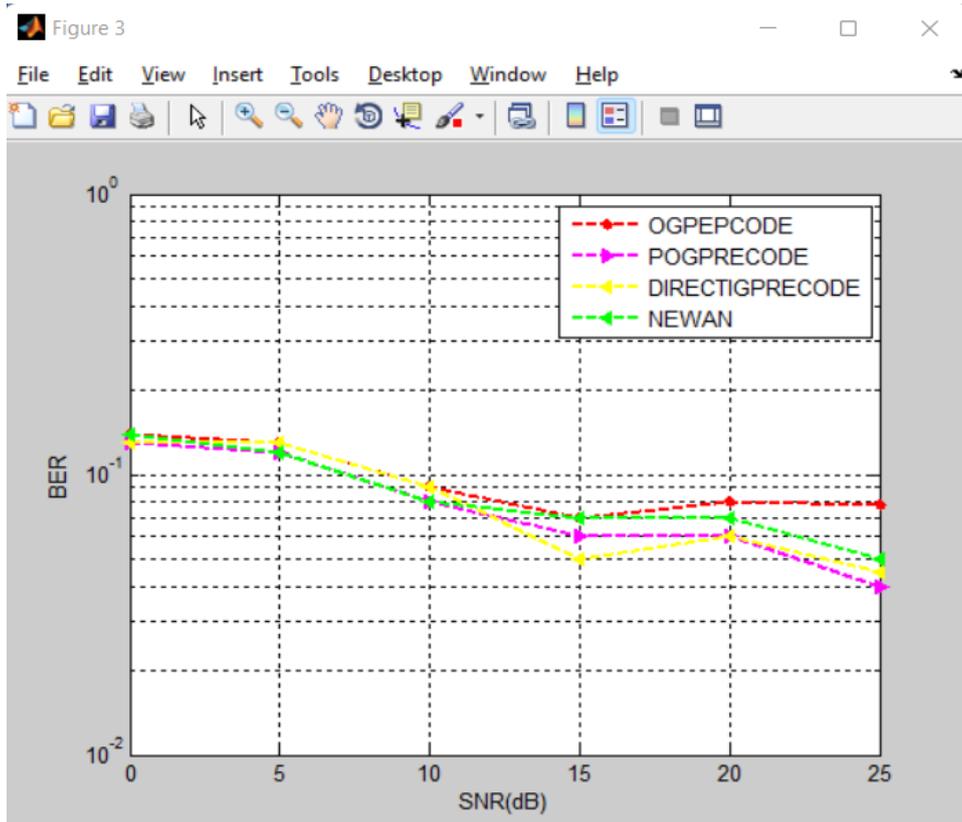
4. RESULTS



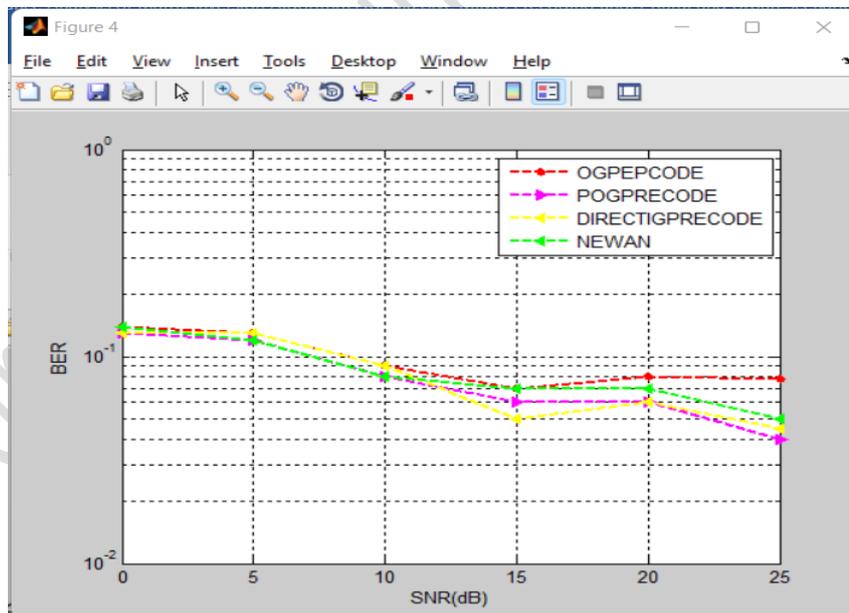
Rural scenario with 100 Tx antennas



Rural scenerios with 200 Tx anten



Urban scenario with 100 Tx antennas



urban scenario with 200 Tx antennas

We have used the different precoding techniques like Conjugate gradient , preconjugate gradient and Neuman series . In the above simulation results, x-axis is bit error rate and y-axis is signal-to-noise ratio. To get efficient output we need to decrease bit error rate and increase signal to noise ratio. Here we took 100 Tx and 200 Tx antennas for comparing rural

5. CONCLUSION

The data symbols of 4 users are simultaneously taken and modulated by 16-QAM modulation and precoded by CG, PCG and Neumann algorithms. These are compared in rural and urban scenarios. From the simulation results we conclude that Neumann series and PCG works well in all type of scenarios for varying SNR values. But, the drawback in Neumann series is the requirement of higher order matrix multiplications as the number of iteration increases and in PCG the selection of suitable preconditioners is much more challenging. Hence, CG based precoding can be selected for both rural micro cell and urban macro cell scenarios since the performance is proven to be better in all scenarios. The implementations of CG based iterative precoding can be simplified by proper selection of methods that accelerate these iterative algorithms so as to reduce the number of computations.

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